Forward Patch Length Design for LV2 Solid Polyethylene Dielectric Antennas

Definitions

Speed of light

```
In[2]:=  vc = c → 2.99792458*^8; (*m/s*)
```

Standard usage (MKS)

In[3]:= $\text{std} = \left\{\epsilon \to \epsilon r\, \epsilon 0,\ \mu \to \mu 0,\ \lambda 0 \to c / f,\ \epsilon 0 \to \frac{1}{\mu 0\, c^2},\ \mu 0 \to 4\,\pi * 1{*}{\wedge}{-7}\right\};$

Assumptions

Geometry, (h) is dielectric thickness, (H) is overall thickness, (w) is patch "length", (a) is the module diameter. Also (ϵr) the dielectric constant,

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In[4]:=  asu = {ϵr → 2.20066, h → 25.4*^-3 * 0.072,
           H → 25.4*^-3 * 0.082, a → 25.4*^-3 * 5.25} (* meters *)

Out[4]=  {ϵr → 2.20066, h → 0.0018288, H → 0.0020828, a → 0.13335}
```

Design target frequencies

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In[5]:=  {fWiFi, fGPS, fATV} = 1*^9 {2.412, 1.57542, 1.25325}(*Hz*);
```

From Harrington, Time Harmonic Electromagnetic Fields p.180-187, the complex admittance of a narrow slot is (Be worried that Harrington gives two answers, see p.183 note 1)

In[6]:= $\text{slotA} = \left\{GL \to \frac{L}{120\,\lambda 0},\ BL \to \frac{L\left(-0.5407541328186911\grave{}\ - 2\,\text{Log}\left[\frac{f\,h}{c}\right]\right)}{120\,\pi\,\lambda 0}\right\};$

Where (GL) is the real part of the admittance, and (BL) is the complex part. (L) is the "width" of the slot, and (h) is the thickness of the dielectric.

The Impedance of a coaxial transmission line is found in Jackson, Classical Electrodynamics p.385, and other references

In[7]:= $ZC0 = Z0 \to \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\,\mathrm{Log}\left[\frac{b}{a}\right]$; (* Ohms *)

In the impedance formula, (a) is the inner diameter of the cylindrical transmission line, and (b) is the outer diameter.

The rule for admittance transformation along a transmission line can be found in any standard reference c.f. Richard C. Johnson ed., Antenna Engineering Handbook 3rd. p.42-6

In[8]:= $At \to 1 \Big/ \left(Z0\,\frac{ZL\,\mathrm{Cos}[\theta] + i\,Z0\,\mathrm{Sin}[\theta]}{Z0\,\mathrm{Cos}[\theta] + i\,ZL\,\mathrm{Sin}[\theta]}\right)$ /. $\left\{ZL \to \frac{1}{AL},\ Z0 \to \frac{1}{A0}\right\}$ // FullSimplify

Out[8]= $At \to \frac{A0\,(AL\,\mathrm{Cos}[\theta] + i\,A0\,\mathrm{Sin}[\theta])}{A0\,\mathrm{Cos}[\theta] + i\,AL\,\mathrm{Sin}[\theta]}$

Derivation

Rewrite the formula for the admittance in terms of electrical length θ

In[9]:= **Aθ = FullSimplify[At /. % /. AL → GL + i BL]**

Out[9]= $\frac{A0\,((i\,BL + GL)\,\mathrm{Cos}[\theta] + i\,A0\,\mathrm{Sin}[\theta])}{A0\,\mathrm{Cos}[\theta] - (BL - i\,GL)\,\mathrm{Sin}[\theta]}$

We wish to transform G + i B → G - i B using a transmission line length θ

In[10]:= **ELL = Solve[GL - i BL == Aθ, θ] // FullSimplify // PowerExpand**

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

Out[10]= $\left\{\left\{\theta \to -\mathrm{ArcCos}\left[-\frac{-A0^2 + BL^2 + GL^2}{\sqrt{A0^4 + 2\,A0^2\,(BL - GL)\,(BL + GL) + (BL^2 + GL^2)^2}}\right]\right\},\right.$

$\left\{\theta \to \mathrm{ArcCos}\left[-\frac{-A0^2 + BL^2 + GL^2}{\sqrt{A0^4 + 2\,A0^2\,(BL - GL)\,(BL + GL) + (BL^2 + GL^2)^2}}\right]\right\},$

$\left\{\theta \to -\mathrm{ArcCos}\left[\frac{-A0^2 + BL^2 + GL^2}{\sqrt{A0^4 + 2\,A0^2\,(BL - GL)\,(BL + GL) + (BL^2 + GL^2)^2}}\right]\right\},$

$\left.\left\{\theta \to \mathrm{ArcCos}\left[\frac{-A0^2 + BL^2 + GL^2}{\sqrt{A0^4 + 2\,A0^2\,(BL - GL)\,(BL + GL) + (BL^2 + GL^2)^2}}\right]\right\}\right\}$

These are all simple reflections about 180°, pick the positive one closest to 180°

In[11]:= **EL = ELL⟦4, 1⟧**

Out[11]= $\theta \to \mathrm{ArcCos}\left[\frac{-A0^2 + BL^2 + GL^2}{\sqrt{A0^4 + 2\,A0^2\,(BL - GL)\,(BL + GL) + (BL^2 + GL^2)^2}}\right]$

EL is a formula for the electrical length (in radians) of a transmission line with real admittance (A0) that transforms input admittance GL + i BL to GL - i BL

Use the EL formula to derive the patch "length"

In[12]:=
```
wCyeq = FullSimplify[
   w == θ λ0 / (2 π Sqrt[ϵr]) /. EL /. slotA /. A0 → 1 / Z0 /. ZC0
    /. {L → π (a + H), b → a + 2 h}
    //. std /. vc]
```

Out[12]=

$$w == \frac{1}{f\sqrt{\epsilon r}}\left(4.77135\times 10^{7}\,\mathrm{ArcCos}\left[\left(7.62598\times 10^{-21}\,f^{2}\,(a+H)^{2}+7.72674\times 10^{-22}\,f^{2}\,(a+H)^{2}\,(-38.4964+2\,\mathrm{Log}[f\,h])^{2}-\frac{0.000278163\,\epsilon r}{\mathrm{Log}[1+\frac{2\,h}{a}]^{2}}\right)\Big/\left(\sqrt{\left(f^{4}\,(a+H)^{4}\,(7.62598\times 10^{-21}+7.72674\times 10^{-22}\,(-38.4964+2\,\mathrm{Log}[f\,h])^{2})^{2}+\frac{7.73744\times 10^{-8}\,\epsilon r^{2}}{\mathrm{Log}[1+\frac{2\,h}{a}]^{4}}+\frac{1}{\mathrm{Log}[1+\frac{2\,h}{a}]^{2}}\,(0.000556325\,f^{2}\,(a+H)^{2}\,\epsilon r\,(9.82759\times 10^{-10}-5.5594\times 10^{-11}\,\mathrm{Log}[f\,h])\,(1.15741\times 10^{-9}-5.5594\times 10^{-11}\,\mathrm{Log}[f\,h]))\right)}\right)\right]\right)$$

Convert the result into a function for w

In[13]:=
```
wcy[a_, ϵr_, h_, H_, f_] :=
  Evaluate[(List @@ wCyeq)〚2〛]
```

Example

Find the patch length at each design frequency

In[14]:=
```
Print["WiFi, GPS, ATV : Patch length (In meters)"];
wdesign =
  Evaluate[wcy[a, ϵr, h, H, #] /. asu] & /@ {fWiFi, fGPS, fATV}
   // EngineeringForm
```

WiFi, GPS, ATV : Patch length (In meters)

Out[15]//EngineeringForm=

$\{39.8032\times 10^{-3},\ 61.8227\times 10^{-3},\ 78.1891\times 10^{-3}\}$

Exposition of formula

In[16]:= **SetPrecision[wCyeq, 10] // TraditionalForm**

Out[16]//TraditionalForm=

$$w == \frac{1}{f^{1.000000000}\,\epsilon\mathrm{r}^{0.5000000000}} \left(4.771345159\times 10^{7} \cos^{-1}\left(\left(\left(7.625983257\times 10^{-21}\, f^{2.000000000}\,(a+H)^{2.000000000} + 7.726736500\times 10^{-22}\, f^{2.000000000}\,(2.000000000\log(f\,h) - 38.49644784)^{2.000000000}\,(a+H)^{2.000000000} - \frac{0.0002781625140\,\epsilon\mathrm{r}}{\log^{2.000000000}\left(\frac{2.000000000\,h}{a^{1.000000000}} + 1.000000000\right)}\right) \Big/ \left(f^{4.000000000}\,(7.726736500\times 10^{-22}\,(2.000000000\log(f\,h) - 38.49644784)^{2.000000000} + 7.625983257\times 10^{-21})\text{^}2.000000000\,(a+H)^{4.000000000} + (0.0005563250280\, f^{2.000000000}\,\epsilon\mathrm{r}\,(9.827591901\times 10^{-10} - 5.559401587\times 10^{-11}\log(f\,h))\,(1.157412942\times 10^{-9} - 5.559401587\times 10^{-11}\log(f\,h))\,(a+H)^{2.000000000}) \Big/ \log^{2.000000000}\left(\frac{2.000000000\,h}{a^{1.000000000}} + 1.000000000\right) + \frac{7.737438420\times 10^{-8}\,\epsilon\mathrm{r}^{2.000000000}}{\log^{4.000000000}\left(\frac{2.000000000\,h}{a^{1.000000000}} + 1.000000000\right)}\right)\text{^}0.5000000000\right)\right)\right)$$