May 1st, 2005 test firing (motor 050105B).

Due to our metering orifice inlet pressure dropping, for a still undetermined reason, we are not able to verify that we have hit our 50lbf target thrust for the gox/paraffin motor. Though it does look like we were operating around 50 lbf thrust for a short period of time. It is difficult to tell what was happening because of the large chamber pressure oscillations during the first 1.5 seconds of motor operation.

To determine if our new larger metering orifice was successful we can attempt to verify it's performance during some other portion of the motor firing where the large combustion chamber oscillations have ceased and the pressure upstream of the metering orifice is fairly constant.

An upstream (metering orifice inlet) pressure value of 769 psi was chosen because the motor had reached as steady state of a condition as it was going to at this point. See Figure 1.

For this point we know the actual metering orifice inlet pressure (768.9 psi) and motor thrust (26.32 lbf) from our data acquisition system. We will verify that our calculations predict the correct motor thrust given the metering orifice inlet pressure.

Our new larger metering orifice (.089") was used in this static firing. Using the following calculation we should be able to determine the mass flow rate of oxygen into the combustion chamber given the metering orifice inlet pressure.

$C_d := .84$ — Flat plate metering orifice

$P_1 := 769\,\text{psi}$ — Upstream pressure

$P_1 = 5.302\times 10^6\,\text{Pa}$

$A_1 := \frac{\pi\cdot(.089\,\text{in})^2}{4}$ — Metering orifice area

$A_1 = 4.014\times 10^{-6}\,\text{m}^2$

$R := 260\frac{\text{J}}{\text{kg}\cdot\text{K}}$ — Gas constant for oxygen

$\gamma := 1.4$ — ratio of specific heats for oxygen

$T_1 := 293.15\,\text{K}$ — Gas temperature

$$q := C_d\cdot A_1\cdot P_1\cdot\sqrt{\frac{\gamma}{R\cdot T_1}}\cdot\left(\frac{2}{1+\gamma}\right)^{\frac{\gamma+1}{2\gamma-2}} \qquad q = 0.044\frac{\text{kg}}{\text{s}}$$

Using a value of 0.044 kg/s as our mass flow rate in our Excel regression rate calculator, it says that if we burned this motor for 7 seconds we would use a total of 0.133 kg of paraffin assuming we maintained that mass flow rate for the entire 7 seconds and the grain burns uniformly.

Excel Spreadsheet excerpt

133.38	grams paraffin at 7 seconds
310.81	grams oxygen at 7 seconds
444.19	total grams
0.98	total lbs
2.33	O:F Ratio

Knowing the mass flow rate, the paraffin mass consumed, a theoretical burn time and an assumed specific impulse for this propellant combination we can calculate what the average thrust generated by the motor would be.

$$m_{dotO2} := .044 \frac{kg}{s}$$

$$m_{paraffin} := .133kg$$ from spreadsheet (mass paraffin consumed at 7 seconds given mdot)

$$T_{burn} := 7s$$

$$Isp := 210s$$

Then

$$m_{O2} := m_{dotO2} \cdot T_{burn}$$

$$m_{O2} = 0.308kg$$

$$m_{propellant} := m_{O2} + m_{paraffin}$$

$$m_{propellant} = 0.441kg$$

Calculate average thrust from burned propellants

$$Thrust := \frac{m_{propellant} \cdot g \cdot Isp}{T_{burn}}$$

$$Thrust = 129.742N$$

$$Thrust = 29.167lbf$$

Therefore if the motor burned uniformly for the 7 seconds at a oxygen mass flow rate of 0.044 kg/s we should expect to see an average thrust of about 29 lbf.

Of course, our static test was not uniform because the metering orifice inlet pressure decayed during the course of the firing. Though it may be safe to assume that during the later part of the firing we can take the average of a range of data points and say that it's indicative of steady state motor performance.

Time (seconds)	Inlet Pressure (psi)	Outlet Pressure (psi)	Thrust (lbf)
3.010	770.15	334.28	27.84
3.012	768.92	330.54	28.07
3.014	770.15	326.80	27.84
3.016	768.92	321.81	27.38
3.018	767.68	319.32	27.15
3.020	770.15	323.06	26.24
3.022	768.92	319.32	26.47
3.024	767.68	313.09	26.24
3.026	770.15	306.85	26.24
3.028	768.92	300.62	25.56
3.030	770.15	300.62	25.56
3.032	770.15	296.88	25.10
3.034	766.45	296.88	25.56
3.036	770.15	293.14	26.01
3.038	767.68	299.37	26.93
3.040	768.92	313.09	26.93
3.042	768.92	323.06	26.93
3.044	767.68	330.54	27.38
3.046	767.68	329.29	27.15
3.048	770.15	325.55	27.15
3.050	767.68	323.06	25.78
3.052	767.68	311.84	24.87
3.054	767.68	305.61	25.33
3.056	770.15	301.87	25.33
3.058	768.92	294.39	24.87
3.060	768.92	293.14	24.41
AVERAGE	768.87	312.85	26.32

Comparing the calculated and measured values:

$$Error_{percent} := \left(1 - \frac{26.32}{29.17}\right) \cdot 100$$

$$Error_{percent} = 9.77$$

We see that they differ from each other by less than 10%. This might indicate that the new metering orifice places our motor design close to our original target. That being said…we may still be off by quite a bit because we don't know for sure the ISP of the propellant and the efficiency of the motor.

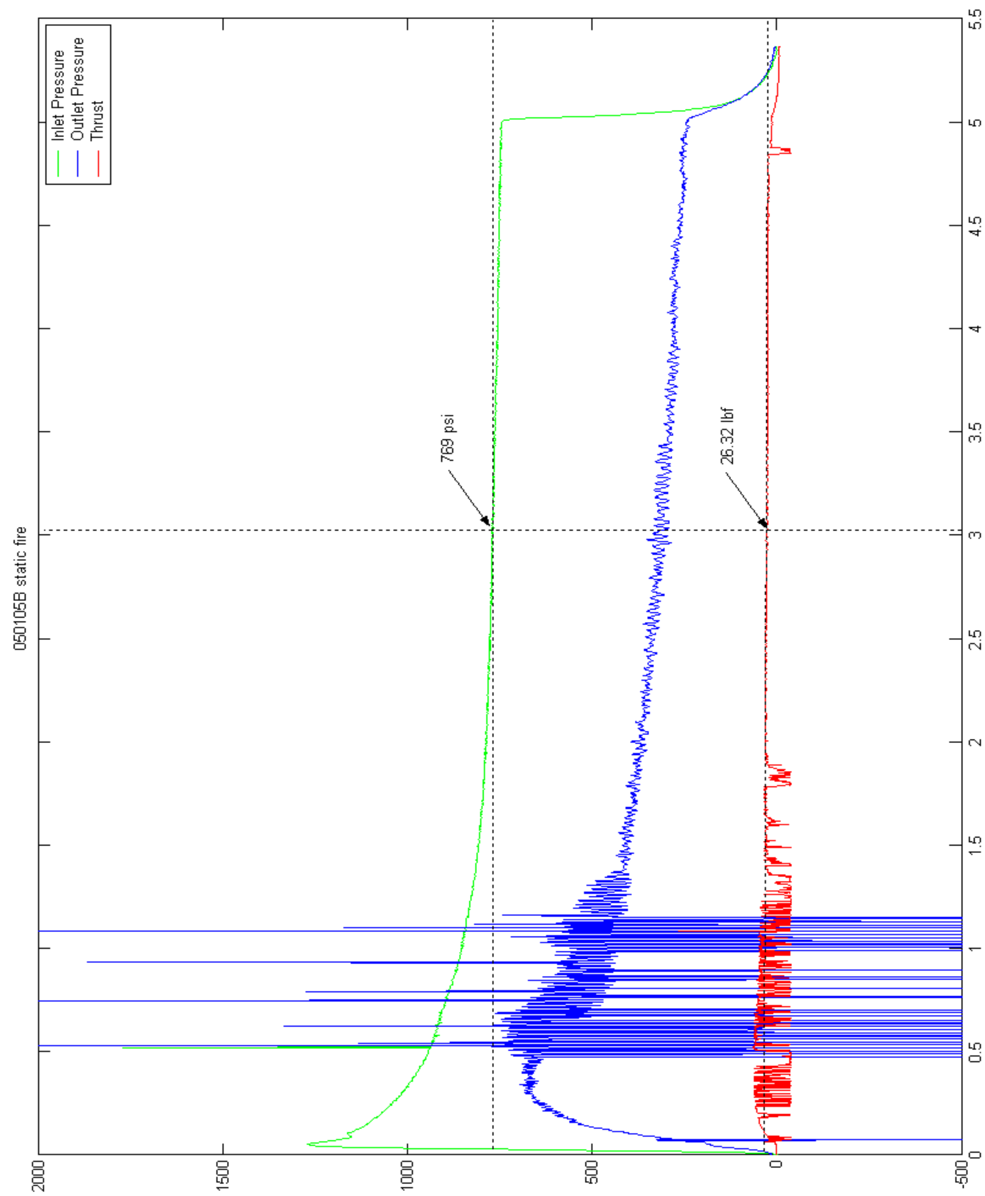

Figure 1