Design of a Flow Metering Orifice for First Wax Hybrid test

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An orifice plate is designed to supply a fixed mass-flow of oxygen to the test motor.

Specifications

The test chamber design pressure is 500 psia, the O_2 inlet supply pressure must be sufficient to cause sonic flow in the orifice. The required mass flow is specified as 35 cubic-feet/minute measured at 15 psia.

First change units and gather some data.

For O_2 the ratio of specific heats (γ) is 1.40

Table 1 gives some properties of various gases at room temperature and pressure.

Gas	Symbol	Molecular Weight	$\gamma = \frac{c_p}{c_v}$	Gas Constant $(R)\left[\frac{J}{kg\,K}\right]$	Specific Heats $c_p\left[\frac{J}{kg\,K}\right]$	$c_v\left[\frac{J}{kg\,K}\right]$	Viscosity $\mu\left[\frac{N\,s}{m^2}\right]$
Air		28.97	1.40	287	1000	716	1.8×10^{-5}
Argon	Ar	39.94	1.67	208	519	310	2.3×10^{-5}
Carbon Dioxide	CO_2	44.01	1.29	189	850	657	1.5×10^{-5}
Carbon Monoxide	CO	28.01	1.40	297	1040	741	1.8×10^{-5}
Helium	He	4.00	1.67	2080	5230	3140	2.0×10^{-5}
Hydrogen	H_2	2.02	1.41	4120	14300	10200	9.1×10^{-5}
Methane	CH_4	16.04	1.32	519	2230	1690	1.1×10^{-5}
Nitrogen	N_2	28.02	1.40	296	1040	741	1.7×10^{-5}
Oxygen	O_2	32.00	1.40	260	913	653	2.0×10^{-5}
Water Vapor	H_2O	18.02	1.33	461	1860	1400	1.1×10^{-5}

Table 1

Using the table, the density of oxygen at room temperature and pressure (NTP, T=293.15°K, p=101325 Pa) can be found from the ideal gas law

$$p = \rho R T \tag{1}$$

For oxygen at room temperature

$$\rho = \frac{p}{R\,T} = \frac{101325\,\text{Pa}}{260\,\frac{\text{J}}{\text{kg K}}\,293.15\,\text{K}} = 1.32939\,\frac{\text{kg}}{\text{m}^3} \tag{2}$$

Assuming that the specification of 15 psia is meant to approximate normal atmospheric pressure. The specified mass flow rate is then

$$35\,\frac{\text{ft}^3}{\text{min}} \Big/ \left(60\,\frac{\text{s}}{\text{min}}\right) \Big/ \left(3.2808\,\frac{\text{ft}}{\text{m}}\right) \times 1.32939\,\frac{\text{kg}}{\text{m}^3} = 21.96\,\text{g}/\text{s} \tag{3}$$

The given pressure in SI units is (reference [2])

$$500\,\text{psia} \times 6894.76\,\frac{\text{Pa}}{\text{psia}} = 3.447\,\text{MPa} \tag{4}$$

■ Background

Since we will fabricate the orifice ourselves the design must be easy to make. Perhaps the easiest plate to make is the sharp-edged concentric orifice. This consists of a round, flat, plate with a single straight sided and un-chamfered hole drilled through the center.

A schematic of the flow through the proposed geometry is shown in figure 1

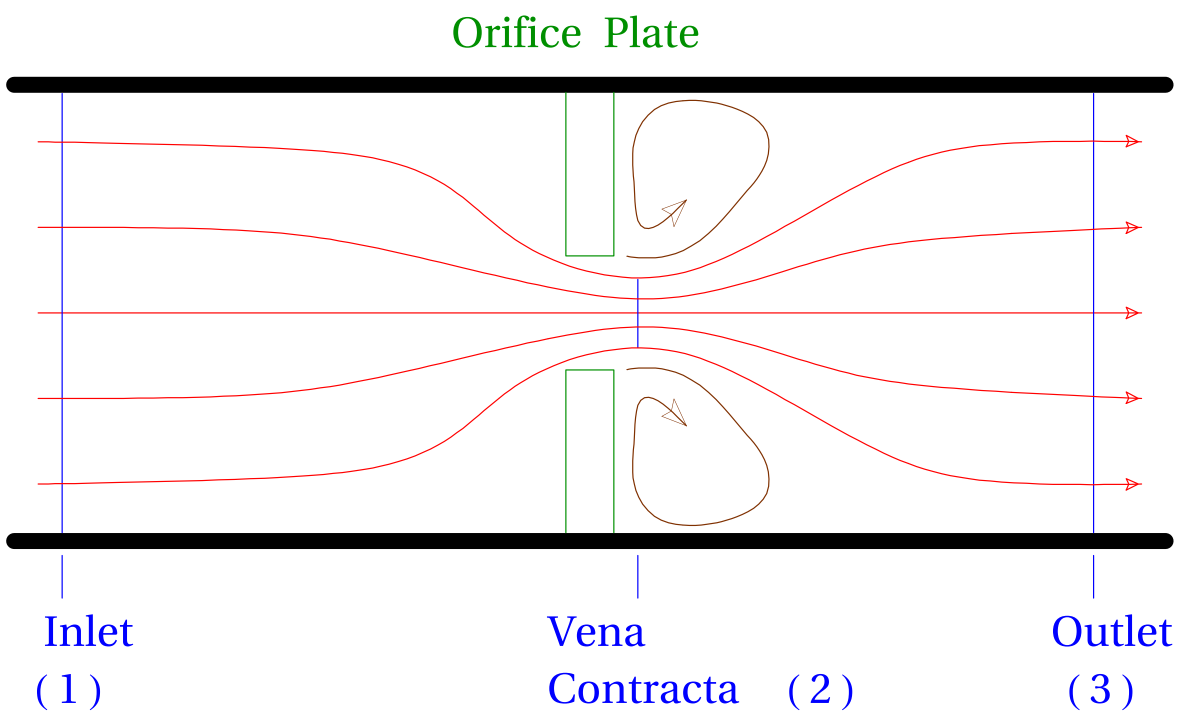

Figure 1

In the figure, station (2) indicates the position where the main flow attains minimum area, minimum pressure, and maximum velocity. This position is known as the vena contracta. Briefly, referring to the figure, the flow enters from the left, and passes station (1) where the pressure and temperature can be approximated as uniform across the pipe. As the flow nears the orifice the stream lines converge and the flow velocity increases while the pressure decreases. Once the flow has passed the orifice plate the stream lines continue to contract for a short distance due to the radially inward momentum imparted to the flow as it enters the plate. This contraction is particularly pronounced for a thin plate. For a very thick plate the flow through the plate approximates flow though a narrow pipe and the vena contracta occurs almost exactly at the end of the plate.

Once the flow leaves the plate it forms a jet that expands into the pipe. In the limit of a pipe diameter that is very much larger than the orifice diameter, the conditions approximate those of a jet flowing into an infinite reservoir and the pressure recovery is zero. For typical flows through a small orifice the pressure recovery is quite low.

The idea behind the orifice as a metering device is to establish a fixed pressure drop between stations (1) and (3), for fixed inlet conditions and pressure drop, the mass-flow through the orifice can be established. To make this work as well as possible the flow rate should vary slowly with respect to the pressure difference. The flow rate can be calculated as the product of the flow velocity times the flow area at any position along the flow. The greatest velocity of flow occurs at station (2) in the figure, the vena contracta, where the flow is narrowest.

If the pressure drop across the orifice is set appropriately, the velocity at station (2) will just reach the local speed of sound at that position. If the pressure difference is slightly increased from this point, the velocity at (2) will not increase (the flow is choked). If the plate is thin compared to the orifice diameter, under just-sonic conditions, the vena contracta will occur fairly far downstream from the end of the plate and its area will be substantially smaller than the area of the orifice. If the pressure at station (3) is further reduced past the just-sonic value, the vena contracta of the thin plate will move upstream and its diameter will increase slightly, therefore the flow rate will also increase slightly. This undesirable variation can be minimized by making the orifice plate thicker, which moves the vena contracta closer to the end of the plate. On the other hand, if the plate is very thick, the orifice cavity behaves like a narrow pipe, then the frictional pipe losses account for a significant part of the pressure drop in the orifice, which makes the flow rate to pressure-difference relationship more complex, also a thick orifice will dissipate more power. There is therefore an optimal range of orifice diameter to plate thickness ratio which provides the most nearly constant flow rate versus pressure difference. This is idea is developed more fully below.

■ Mathematical Description

Assume the flow is cylindrically symmetric about the centerline of the pipe. Define cylindrical coordinates. Let z be the coordinate along the pipe axis, increasing to the right, the origin being at the midpoint of the orifice plate. Let the radial distance from the centerline be r and the angle about the center line be θ. All parameters of interest are constant with respect to θ by symmetry. Let the pipe diameter be capital D, while the orifice diameter is lowercase d. The plate thickness is given by t. An incomplete table of symbols appears below.

Description	Symbol	Units
Area	A	m^2
Sonic Velocity	a	m / s
Discharge Coefficient	C_d	dimensionless
Orifice Diameter	d	m
Pipe Inside Diameter	D	39.94
Mach Number	M	dimensionless
Pressure	p	Pa
Mass Flow Rate	q	kg / s
Orifice Plate Thickness	t	m
Temperature	T	K
Velocity Along Pipe Axis	u	m / s
Distance Along Pipe Axis	z	m
Ratio of Specific Heats	γ	dimensionless
Angle about Pipe Axis	θ	radians
Mass Density	ρ	kg/m^3
Stress in the plate	σ	Pa

Table 2

Again refer to figure 1, assume that the contraction of the flow from station (1) to (2) is isentropic, further assume that the flow is of a compressible perfect gas, and that the gravitational potential energy is negligible. Under these assumptions, the total (stagnation) enthalpy is a constant of the flow. Computing this at station (1) gives

$$T_t == T_1 \left(1 + \frac{\gamma - 1}{2} M_1{}^2\right) \tag{5}$$

Where T_1, and M_1 indicate the local temperature and Mach number at station (1).

Since we assume that the Mach number at station (2) is equal to one, the temperature at (2) can be found in terms of the conditions at station (1).

$$\begin{gathered} T_2 == T_t \Big/ \left(1 + \frac{\gamma - 1}{2} M_2{}^2\right) == \\ T_1 \left(1 + \frac{\gamma - 1}{2} M_1{}^2\right) \Big/ \left(1 + \frac{\gamma - 1}{2}\right) == 2\, T_1 \left(1 + \frac{\gamma - 1}{2} M_1{}^2\right) \Big/ (\gamma + 1) \end{gathered} \tag{6}$$

Because the flow is isentropic and the gas is perfect, a simple relationship holds between the pressure and temperature at any two points.

$$\frac{p_2}{p_1} == \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{7}$$

From this, the pressure at station (2) is found to be

$$p_2 == p_1 \left(2\, T_1 \, \frac{1 + \frac{\gamma-1}{2} M_1{}^2}{T_1\, (\gamma+1)}\right)^{\frac{\gamma}{\gamma-1}} == p_1 \left(\frac{2 + (\gamma-1)\, M_1{}^2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \tag{8}$$

The mass flow rate can be computed at station (2), where the velocity is known to be sonic.

$$q == A_2\, a_2\, \rho_2 \tag{9}$$

The chief difficulty remaining is that the area of the vena contracta A_2 is not known. From the work in reference [1] it can be assumed that the area A_2 differs from the actual area of the orifice A, by a known constant factor, namely

$$C_d == A_2 / A \tag{10}$$

The constant C_d is known as the coefficient of discharge. According to [1] the value of C_d ranges from about 0.81 to 0.86 with a mean of about 0.84. For the discharge coefficient to maintain its specification the orifice plate aspect ratio (t/d) must be approximately in the range between 1 and 7, and the pressure ratio should be sufficient to make the flow sonic at station (2). It's probably true that pressure ratios much greater that the sonic ratio can also introduce errors, but no data is in hand.

Using the discharge coefficient, the mass flow rate is given as

$$q == C_d\, A\, a_2\, \rho_2 == C_d\, A \sqrt{\gamma\, R\, T_2}\, \frac{p_2}{R\, T_2} == C_d\, A \sqrt{\frac{\gamma}{R\, T_2}}\, p_2 \tag{11}$$

In terms of conditions at station (1)

$$\begin{aligned} q &== C_d\, A \sqrt{\frac{\gamma}{R\, T_2}}\, p_2 == \\ &\mathrm{Cd}\, A\, p_1 \left(\frac{2 + (\gamma-1)\, M_1^2}{1+\gamma}\right)^{\frac{\gamma}{-1+\gamma}} \sqrt{\frac{\gamma\,(1+\gamma)}{R\left(2 + (\gamma-1)\, M_1^2\right) T_1}} \\ &== \mathrm{Cd}\, A\, p_1 \sqrt{\frac{\gamma}{R\, T_1}} \left(\frac{2 + (\gamma-1)\, M_1^2}{1+\gamma}\right)^{\frac{\gamma+1}{2\gamma-2}} \end{aligned} \tag{12}$$

The pressure ratio required for sonic flow can be found from equation (8)

$$\frac{p_2}{p_1} == \left(\frac{2 + (\gamma - 1) M_1{}^2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} \tag{13}$$

Assuming M_1 is small, and oxygen as the working fluid gives

$$\frac{p_2}{p_1} \approx \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}, \gamma \to 1.4 = 0.528 \tag{14}$$

So if p_2 is equal to 500 psia, p_1 must be greater than 947 psia for sonic flow.

Design

The mass flow is specified, use equation (12) to find the orifice area. The assumptions made are listed in the table below.

Description	Symbol	Assumed Value
Discharge Coefficient	C_d	0.84 ± 0.03
Pipe Inside Diameter	D	large compared to orifice diameter
Mach Number (1)	M_1	small compared to unity
Pressure station (1)	p_1	1000 psia = 6894760 Pa
Mass Flow Rate	q	21.96 g / s
Orifice Plate Thickness	t	between 1 and 7 orifice diameters
Temperature (1)	T_1	293.15 K
Ratio of Specific Heats	γ	1.4 (O_2)

Table 3

$$A == q \frac{\left(\frac{2+(\gamma-1) M_1^2}{1+\gamma}\right)^{\frac{\gamma+1}{2-2\gamma}}}{Cd\, p_1 \sqrt{\frac{\gamma}{R\, T_1}}} \approx q \frac{\left(\frac{2}{1+\gamma}\right)^{\frac{\gamma+1}{2-2\gamma}}}{Cd\, p_1 \sqrt{\frac{\gamma}{R\, T_1}}} = 1.52878 \times 10^{-6}\ m^2 \tag{15}$$

The orifice diameter is therefore 1.395 mm = 0.0549 in.

The range of acceptable orifice plate thickness is 0.055 - 0.384 in. In particular this includes 1/8", 3/16" and 1/4" plate. 1/8" plate provides an aspect ratio of about 2.3, which is near the geometric center of the range. For safety it is desirable that the orifice plate be able to support the full line pressure, in this case 1000 psia. Assume a 3/4" inside diameter pipe, the largest likely to be encountered in this apparatus,

Find the deflection of a simply supported plate with uniform pressure load (See EN175 notes, reference [3])

For a simply supported disk, the tensile yield stress of a ductile material should be greater than $p\, D^2 / (4\, t^2)$. Assume the maximum diameter is 3/4" then the strength requirement is

$$\sigma > \frac{p_1 D^2}{4 t^2} = \frac{625.531}{t^2} \text{ N} \tag{16}$$

Here N indicates Newtons

For brass the tensile yield is about 137 MPa (varies this is a low value), for mild steel (alloy 1020) it’s 206 MPa (easily double this for strong steel), and for aluminum 6061−T6 it is about 213 MPa.

Picking 200 MPa for soft steel or hard aluminum, and sticking with 137 MPa for brass, and applying a safety factor of 3, the safe thicknesses are 0.145" for brass and 0.121" for soft steel/hard aluminum. Either one of these choices gives a fairly good aspect ratio for the orifice.

■ Summary

This design should be successful. Factors not accounted for include velocity at the inlet (M_1 assumed << 1), losses in the plumbing system, and variation in inlet temperature (gas arrives after expansion from a high pressure cylinder, so may be lower than room temperature). The nearest standard drill to the specified size is a #54 with probable oversize the hole will be ~0.0565 in. The orifice will operate best with a some clear distance up and down stream. Ideally more than 2 pipe diameters up and 3 pipe diameters down stream, minimally 1 diameter each way. Keeping the pipe large in comparison to the orifice is advisable, 1/2" seems reasonable, 1/4" at minimum.

■ References

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