Given:

- Both static and dynamic (fatigue) failure criteria will be used.
- A minimum factor of safety =2 will be adhered to.
- For fatigue analysis the ASME elliptic model with Von Mises equivalent stress will be used

Assumptions:

- Chamber pressure = 500 psi
- Maximum chamber pressure fluctuations = 25% of mean chamber pressure [1]
- Thermal effects will initially be neglected because of short burn times and insulative properties of fuel grain (Temperature factor = 1)
- Loading is purely axial. (Size factor =1)

Find:

1) Pressure vessel analysis: (See Figure 1.0)

- Combustion chamber material is 6061 T6 aluminum OD = 2.35" Length = 8.00" THK= 0.140"
- Only use thin wall assumption if radius to thickness ratio = 20:1 [2]
- For static analysis find maximum tensile and shear stressed in plane and out of plane. *Calculate safety factor using given material properties
- For dynamic analysis use Von Mises equivalent stress with ASME elliptic
- Sigma alternating = 25% sigma mean [1]
- Find corrected endurance limit using correction coefficients as applicable to [2].

2) Forward bolted connection analysis: (See Figure 1.0)

- Material #1 = SS 316L annealed plate (0.205 inches thick)
- Material #2 = 6061 T6 aluminum (0.1875 inches thick)
- Assume reusable connection (75% of proof load)
- Use static and dynamic situations
- Use both bolt yielding and joint separation failure criteria
- Fatigue analysis – Von Mises equivalent stress, ASME elliptic, S.F. =2
- Assume bolts extend from nuts 2 threads
- No shanks on bolts

3) Aft bolted connection analysis: (See Figure 1.0)

- Material #2 = 6061 T6 aluminum (0.1875 inches thick)
- Material #2 = 6061 T6 aluminum (0.1875 inches thick)
- Assume reusable connection (75% of proof load)
- Use static and dynamic situations
- Use both bolt yielding and joint separation failure criteria
- Fatigue analysis – Von Mises equivalent stress, ASME elliptic, S.F. =2
- Assume bolts extend from nuts 2 threads
- No shanks on bolts

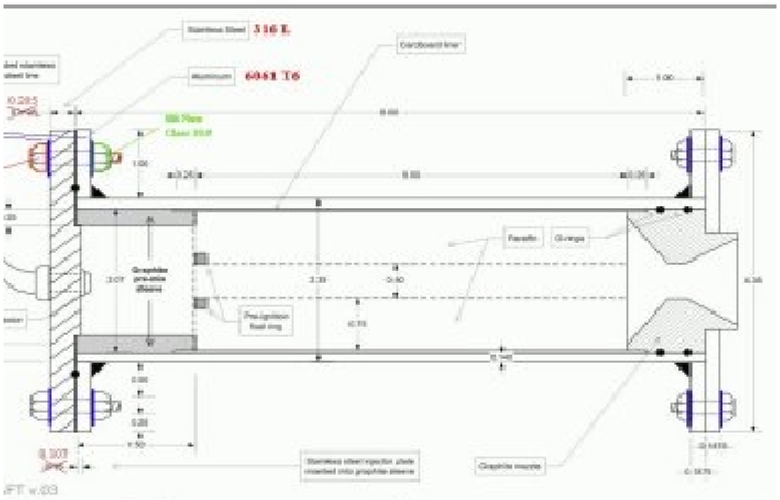
4) Flange to chamber weld analysis: (See Figure 1.0)

- Coming soon! Practicing TIG welding and learning more about weld quality.

References:

[1] NASA /TP -2000-209905
[2] "Mechanical Engineering Design", Shigley & Meschke 5th ed.

Figure :



Solution :

1) The first analysis will be on the pressure vessel. The motor casing will be modeled as a cylindrical pressure vessel. The ratio of the radius of the pressure vessel to the thickness of the wall is 8.4 which is below the threshold for the vessel to be considered thin walled.

The tangential stress will be measured at the inner and outer surfaces and the radial stress will be measured at the inner surface where it it highest. The longitudinal stress also is computed and is assumed to be constrant through the thickness of the wall.

The following equations for Tangential (σt) and Radial (σr) Stress are taken from [2]

$p_i := 500psi \quad p_o := 14.7psi \quad r_o := 1.175in \quad r_i := 1.105in \quad thickness := 0.140in$

$$\sigma_{t.i} := \frac{p_i \cdot r_i^{\,2} - p_o \cdot r_o^{\,2} - r_i^{\,2}\, r_o^{\,2} \frac{(p_o - p_i)}{r_i^{\,2}}}{r_o^{\,2} - r_i^{\,2}}$$

$$\sigma_{t.o} := \frac{p_i \cdot r_i^{\,2} - p_o \cdot r_o^{\,2} - r_i^{\,2}\, r_o^{\,2} \frac{(p_o - p_i)}{r_o^{\,2}}}{r_o^{\,2} - r_i^{\,2}}$$

$$\sigma_{r.i} := \frac{p_i \cdot r_i^{2} - p_o \cdot r_o^{2} + r_i^{2}\, r_o^{2}\, \frac{(p_o - p_i)}{r_i^{2}}}{r_o^{2} - r_i^{2}}$$

$$\sigma_{long} := \frac{p_i \cdot r_i^{2}}{r_o^{2} - r_i^{2}}$$

Returning the values;

$\sigma_{t.i} = 7.896 \times 10^{3}\ \text{psi}$ $\quad$ $\sigma_{t.o} = 7.411 \times 10^{3}\ \text{psi}$ $\quad$ $\sigma_{r.i} = -500\ \text{psi}$ $\quad$ $\sigma_{long} = 3.825 \times 10^{3}\ \text{psi}$

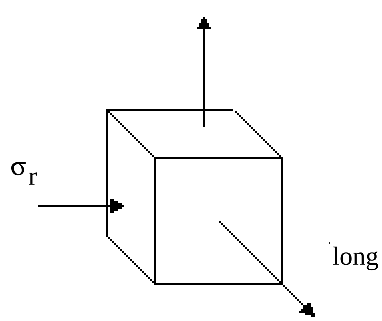
The thin walled analysis taken from James M. Gere: Mechanics of Materials 5th addition, returns;

$$\sigma_{circumferential} := \frac{p_i \cdot r_i}{\text{thickness}} \qquad \sigma_{longitudinal} := \frac{p_i \cdot r_i}{2 \cdot \text{thickness}}$$

$$\boxed{\sigma_{circumferential} = 3.946 \times 10^{3}\ \text{psi}}$$

$$\boxed{\sigma_{longitudinal} = 1.973 \times 10^{3}\ \text{psi}}$$

The analysis taking into account the thickness of the wall gives the following stressed element found at the inside of the cylinder wall.



The stresses in the analysis are without shear and are therefore the principal stresses acting on that stress componenet. This perhaps ignores the shear stress that is encountered due to the variance in σ_t as r increases. However, As this element is rotated shear stresses will act on the element. This can be easily pictured with a Mohr cricle diagram and will give the maximum shear shown below.

$$\tau_{max.cylinder} := \frac{\sigma_{t.i} - \sigma_{r.i}}{2}$$

$$\tau_{max.cylinder} = 4.198 \times 10^3 \, psi$$

Static Analysis of Cylinder

A factor of safety will be given for the maximum normal stress, maximum shear stress and the Von Mises stress with the criteria for failure being yeilding.

The Von Mises stress is given below.

$$\sigma_{vm.c} := \sqrt{\frac{\left(\sigma_{t.i} - \sigma_{r.i}\right)^2 + \left(\sigma_{r.i} - \sigma_{long}\right)^2 + \left(\sigma_{t.i} - \sigma_{long}\right)^2}{2}}$$

$$\sigma_{vm.c} = 7.272 \times 10^3 \, psi$$

The following material properties are taken from Matweb.com for Aluminum 6061

$$S_{ut.al} := 45000psi \qquad S_{y.al} := 39900psi \qquad S_{sy.al} := 0.5 \cdot S_{y.al}$$

$$N_{c.s.\sigma} := \frac{S_{y.al}}{\sigma_{t.i}} \qquad N_{c.s.\tau} := \frac{S_{sy.al}}{\tau_{max.cylinder}} \qquad N_{c.s.vm} := \frac{S_{y.al}}{\sigma_{vm.c}}$$

$$N_{c.s.\sigma} = 5.053$$

$$N_{c.s.\tau} = 4.752$$

$$N_{c.s.vm} = 5.486$$

Fatigue Analysis of Cylinder

The cylinder is to have a mean pressure of 500 psi with an alternating pressure equal to 25% of the mean. The ASME elliptic criteria will be used. The Von Mises Stress for the mean pressure was calculated above. Using the same procedure a for the alternating stress of 125 psi a Von Mises stress 1658 psi.

$$\sigma_{vm.c.a} := 1658psi$$

Aluminum doesn't have a clear endurance limit, however matweb.com gives a fatigue strength value of 13,800 psi based on $5x10^8$ cycles

$$S_{e.uc} := 13800psi$$

Finding the corrected endurance limit requires determining the Surface, Size, Load, Temperature, and Reliability factors.

- Assuming the surface of the cylinder is of machined quality Shigley Gives the following for determining The surface factor $K_{a.}$

$$a_c := 2.7 \qquad b_c := -0.265$$

$$K_{a.c} := a_c \cdot (45)^{b_c}$$

$$K_{a.c} = 0.985$$

- For axial loading there is no size effect, therefore the size factor is equal to one.

$$K_{b.c} := 1$$

- Shigley gives the load factor of 0.923 for materials with an ultimate strength below 220 kpsi loaded axially

$$K_{c.c} := 0.923$$

- The temperature factor is dependent on the temperature of the vessel. The characteristic of the burn time is such that the parafin grain will insulate the walls of the cylinder allowing a temperature factor of 1 to be used.

$$K_{d.c} := 1$$

- Norton recognizes the scatter of data that was used to obtain the above factors and introduces a reliability factor to account for this. For 99% reliability a factor of 0.814 has been determined.

$$K_{e.c} := 0.814$$

The endurance strength is calculated with 99% reliability to be;

$$S_{e.c} := S_{e.uc} \cdot K_{a.c} \cdot K_{b.c} \cdot K_{c.c} \cdot K_{d.c} \cdot K_{e.c}$$

$$S_{e.c} = 1.021 \times 10^{4}\,\mathrm{psi}$$

Using the ASME Elliptic the factor of safety is determined for fatigue.

$$\left(N_{c.f} \cdot \frac{\sigma_{vm.c.a}}{S_{e.c}}\right)^2 + \left(N_{c.f} \cdot \frac{\sigma_{vm.c}}{S_{ut.al}}\right)^2 = 1$$

Solving for $N_{c.f}$

$$N_{c.f} := \frac{S_{ut.al} \cdot S_{e.c}}{\sqrt{\sigma_{vm.c.a}{}^2 \cdot S_{ut.al}{}^2 + \sigma_{vm.c}{}^2 \cdot S_{e.c}{}^2}}$$

$$N_{c.f} = 4.365$$

Summary of Pressure Vessel:

The pressure vessel described aboave meets or exceeds the desired factor of safety for the stated static and dynamic loading. A summary of the key values is given below.

Tangential stress at inner wall:	$\sigma_{t.i} = 7.896 \times 10^{3}\,psi$
Tangential stress at outer wall	$\sigma_{t.o} = 7.411 \times 10^{3}\,psi$
Radial Stress at inner wall	$\sigma_{r.i} = -500\,psi$
Longitudinal stress at inner wall	$\sigma_{long} = 3.825 \times 10^{3}\,psi$
Von Mises stress at inner wall	$\sigma_{vm.c} = 7.272 \times 10^{3}\,psi$
Alternating Von Mises stress	$\sigma_{vm.c.a} = 1.658 \times 10^{3}\,psi$
Corrected Endurance Limit of Al 6061	$S_{e.c} = 1.021 \times 10^{4}\,psi$
Factor of Safety for Static Stress	$N_{c.s.\sigma} = 5.053$
Factor of Safety for Static Shear	$N_{c.s.\tau} = 4.752$
Factor of Safety for Static Von Mises Stress	$N_{c.s.vm} = 5.486$
Factor of Safety for fatigue analysis	$N_{c.f} = 4.365$

Variables

Mean Pressure of Cylinder	$p_i = 500\ \text{psi}$
Atmospheric Pressure	$p_o = 14.7\ \text{psi}$
Major Radius of Vessel	$r_o = 1.175\ \text{in}$
Minor radius of Vessel	$r_i = 1.105\ \text{in}$
Wall thickness of Vessel	$\text{thickness} = 0.14\ \text{in}$
Tangential stress at inner wall	$\sigma_{t.i} = 7.896 \times 10^3\ \text{psi}$
Tangential stress at outer wall	$\sigma_{t.o} = 7.411 \times 10^3\ \text{psi}$
Radial stress at inner wall	$\sigma_{r.i} = -500\ \text{psi}$
Longitudinal stress of vessel	$\sigma_{long} = 3.825 \times 10^3\ \text{psi}$
Maximum Shear at inner wall	$\tau_{max.cylinder} = 4.198 \times 10^3\ \text{psi}$
Von Mises Stress at inner wall	$\sigma_{vm.c} = 7.272 \times 10^3\ \text{psi}$
Alternating Von Mises Stress	$\sigma_{vm.c.a} = 1.658 \times 10^3\ \text{psi}$
Corrected Endurance Strength	$S_{e.c} = 1.021 \times 10^4\ \text{psi}$

Safety Factors

Static stress of cylinder $N_{c.s.\sigma} = 5.053$

Static shear of cylinder $N_{c.s.\tau} = 4.752$

Static Von Mises of cylinder $N_{c.s.vm} = 5.486$

Material Properties

Aluminum : $S_{ut.al} = 4.5 \times 10^4 \, psi$

$S_{y.al} = 3.99 \times 10^4 \, psi$

$S_{sy.al} = 1.995 \times 10^4 \, psi$