Best estimates for X and Y given X, Y and Q

In the LV2 imu we measure the xy acceleration and redundantly measure a quadrature "q" axis.

Using the redundant measurements in combination

The q axis is oriented in the xy−plane, angled $45°$ from the positive x and y axes. The q axis is parallel to the unit vector $< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$.

The projection of <xy> onto the q−axis is

$$q == \langle x, y \rangle \cdot \langle 1, 1 \rangle \big/ \sqrt{2} == \frac{x+y}{\sqrt{2}} \tag{1}$$

Together xyq constitute a redundant measurement of two independent variables. If the independent variables are taken directly as x and y, what values of x and y are the most likely, given the three noisy measurements xyq?

Symbolically, there are functions such that

$$\hat{x} == f[\tilde{x}, \tilde{y}, \tilde{q}], \quad \hat{y} == g[\tilde{x}, \tilde{y}, \tilde{q}]$$

Where '~' indicates measured values and '^' estimated values.

Considering just $\hat{x}$, we make two independent estimates

$$\hat{x}_1 == \tilde{x}, \quad \hat{x}_2 == \sqrt{2}\,\tilde{q} - \tilde{y} \tag{2}$$

The accelerometers closely follow the Gaussian noise model characterized by standard deviation (σ).

Since $\hat{x}_2$ is a linear function of Gaussian variables it is also a Gaussian variable.

The best estimate for the true value of x given two measurements with Gaussian statistics is given by this well known rule ("well known" ≡ we're too lazy to explain)

$$\hat{x} == \frac{\tilde{x}\,\sigma_{x2}^2}{\sigma_x^2 + \sigma_{x2}^2} + \frac{\sigma_x^2\,\hat{x}_2}{\sigma_x^2 + \sigma_{x2}^2} \tag{3}$$

The variance (standard deviation squared) of $\hat{x}_2$ is given by (4)

$$
\begin{aligned}
\sigma_{x2}{}^{2} &== \sigma_{y}{}^{2}\,(\partial_{y}\,x)^{2} + \sigma_{q}{}^{2}\,(\partial_{q}\,x)^{2} \\
&== 2\,\sigma_{q}^{2} + \sigma_{y}^{2}
\end{aligned}
\tag{4}
$$

Inserting the above formula for variance into equation (3) the resulting estimate is

$$
\hat{x} == \frac{\left(\sqrt{2}\,\tilde{q} - \tilde{y}\right)\sigma_{x}^{2} + \tilde{x}\left(2\,\sigma_{q}^{2} + \sigma_{y}^{2}\right)}{2\,\sigma_{q}^{2} + \sigma_{x}^{2} + \sigma_{y}^{2}}
\tag{5}
$$

Using q as part of an optimal estimate involves only a couple adds and multiplies, assuming constant variances, everything else can be pre−computed.

Here is one possibility using 2 adds and 3 multiplies

$$
\begin{aligned}
s_{1} &== 2\,\sigma_{q}^{2} + \sigma_{y}^{2} + \sigma_{x}^{2} \\
f_{1} &== \left(2\,\sigma_{q}^{2} + \sigma_{y}^{2}\right)/\,s_{1} \\
f_{2} &== \sigma_{x}^{2}\,/\,s_{1} \\
\hat{x} &== \tilde{x} * f_{1} + \left(\sqrt{2} * \tilde{q} - \tilde{y}\right) * f_{2}
\end{aligned}
\tag{6}
$$

Consistency and missing measurements

Since the xyq measurements are redundant, they will usually be slightly inconsistent due to measurement errors. If the measurements were perfectly consistent then they would agree exactly

In practice, the two estimates of x should be close together.

$$
\hat{x}_{1} \to \tilde{x} \approx \hat{x}_{2} \to \sqrt{2}\,\tilde{q} - \tilde{y}
\tag{7}
$$

Due to various conceivable faults it is possible that any of the three sensor outputs could unexpectedly become invalid. Although it is unlikely that random failure will occur during flight, it is moderately likely that rapid motions or vibration could temporarily saturate a sensor. If random failure did occur in only one sensor, the remaining measurements still provide both x and y values, with some loss of precision. The difficulty in this circumstance is in detecting the invalid output. Not enough information about the xyq measurements has been given here to eliminate a bad sensor output, so some outside knowledge must be applied. The easiest thing to do is determine the dynamic range of the accelerometers and detect saturation. If one sensor saturates, its measurement can be eliminated from the computation.

More elaborate schemes probably require an on−line variance estimate. The basic idea being, once a measurement variance exceeds a threshold, the measurement is thrown out, or at least barely heeded, until the variance again falls below a threshold.